

10/31/19

MIS8 (Continued)

Multiple (Independent) Lives - Single Decrement

$${}_n P_{xy} = {}_n P_x \cdot {}_n P_y$$

$${}_n \bar{b}_{xy} = {}_n \bar{b}_x \cdot {}_n \bar{b}_y$$

$$\mu_{xy}(t) = \mu_x(t) + \mu_y(t)$$

$${}_n \bar{b}_{xy} = \int_0^n {}_t P_{xy} \cdot \mu_{xy}(t) dt$$

$${}_n \bar{b}_{xy}^2 = \int_0^n {}_t \bar{b}_{xy} \cdot {}_t P_x \cdot \mu_x(t) dt$$

Exercises:

1) }
 2(a) } See 10/22 notes
 2(h) }

2(i): mortality for (40) is DML ($w = 120$) $\rightarrow {}_n P_x = \frac{w-x-n}{w-x}$
 (45) is LF ($\mu = 0.025$) $\rightarrow {}_n P_x = e^{-\mu \cdot n}$

$${}_{20} \bar{b}_{40:45} = 1 - {}_{20} P_{40:45} = 1 - {}_{20} P_{40}^m \cdot {}_{20} P_{45}^f = 1 - \frac{60}{80} \cdot e^{-.5}$$

$$\begin{aligned}
 2(j): {}_{20} \bar{b}_{40:45} &= \int_0^{20} {}_t P_{40:45} \cdot \mu_{40}(t) \cdot dt \\
 &= \int_0^{20} {}_t P_{45} \cdot \underbrace{{}_t P_{40}}_{= f_{40}(t) \text{ male}} \cdot \mu_{40}(t) dt \\
 &= \int_0^{20} {}_t P_{45} \cdot \frac{1}{80} \cdot dt
 \end{aligned}$$

$$\therefore {}_{20} \bar{b}_{40:45} = \frac{1}{80} \int_0^{20} e^{-.025t} dt = \frac{1}{80} \cdot \frac{1}{.025} \left(e^{-.025 \cdot 0} - e^{-.025 \cdot 20} \right) = \frac{1}{2} (1 - e^{-.5})$$

2(k): Note $\underbrace{{}_{20}\ddot{a}_{40:\overline{45}|}}_{\#2(i)} = \underbrace{{}_{20}\ddot{a}_{40:\overline{45}|}^1}_{\#2(j)} + \underbrace{{}_{20}\ddot{a}_{40:\overline{45}|}^2}_{\#2(k)}$

$$\begin{aligned} \therefore {}_{20}\ddot{a}_{40:\overline{45}|}^1 &= \left(1 - \frac{3}{4}e^{-.5}\right) - \frac{1}{2}(1 - e^{-.5}) \\ &= \frac{1}{2} - \frac{1}{4}e^{-.5} \end{aligned}$$

More Directly, we get

$$\begin{aligned} {}_{20}\ddot{a}_{40:\overline{45}|}^1 &= \int_0^{20} {}_tP_{40}^m \cdot {}_tP_{45}^f \cdot \mu_{45}(t) dt \\ &= \int_0^{20} \frac{80-t}{80} \cdot e^{-.025t} \cdot (.025) dt \\ &\stackrel{\text{IBP}}{=} \frac{1}{2} - \frac{1}{4}e^{-.5} \end{aligned}$$

2(n) ${}_{10}\ddot{a}_{40:\overline{45}|}^2 = \int_0^{10} {}_t\ddot{a}_{40} \cdot \underbrace{{}_tP_{45}}_{\text{does not simplify}} \cdot \mu_{45}(t) dt$

$$\therefore \overline{{}_{10}\ddot{a}_{40:\overline{45}|}^2} = \#2(l)$$

$$\therefore \overline{{}_{10}\ddot{a}_{40:\overline{45}|}^2} = \#2(m)$$

Then ${}_{10}\ddot{a}_{40:\overline{45}|}^2 = \#2(l) - \#2(m)$

H.S) (a) ${}_tP_{xx} = ({}_tP_x)^2 \frac{DML(w)}{w-x} \left(\frac{w-x-t}{w-x}\right)^2$ survival function
 (b) for single life single decrement GDMML($w, \alpha=2$) model

$$(c) \quad \overset{\circ}{e}_{\overline{xx}} = \overset{\circ}{e}_x \text{ for a GDMML } (w, \alpha=2)$$

$$= \frac{w-x}{\alpha+1} = \frac{w-x}{3}$$

$$(d) \quad \overset{\circ}{e}_{\overline{xx}} = \overset{\circ}{e}_x + \overset{\circ}{e}_x - \overset{\circ}{e}_{xx}$$

$$= \frac{w-x}{2} + \frac{w-x}{2} - \frac{w-x}{3} = \frac{2}{3}(w-x)$$

At this time omit #7 & #8.